

# An Analytical Approach to Characterize the Service Process and the Tradeoff between Throughput and Service Time Burstiness in IEEE 802.11 DCF

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**Abstract**—We derive a characterization of the probability distribution of the service time process in a saturated IEEE 802.11 wireless LAN under DCF MAC protocol, both from the point of view of a single station and of the system as a whole. Our service time distribution model is then exploited to highlight the burstiness of service times and its dependence on the maximum value of the IEEE 802.11 contention window. We discuss a trade-off between throughput efficiency and service time variance, showing that a minor throughput loss can bring about a major benefit in service time smoothness, at least for practical values of the number of simultaneously competing stations (e.g. less than 15).

## I. INTRODUCTION

Performance evaluation of a single-hop, Independent Basic Service Set (IBSS) IEEE 802.11 Distributed Coordination Function (DCF) has been largely focused on average, long-term metrics, like saturation throughput (e.g. see [1], [2], [3], [8]), non saturated average throughput (e.g. [9]), delay analysis (e.g. [10] and [11]), average throughput of long-lived TCP connections (e.g. [12] and [13]) and short-lived TCP connections (e.g. [14]).

We aim at characterizing the 802.11 DCF from an external point of view, i.e. as a server of upper layer data units. To this end we focus on an IBSS made up of  $n$  stations, possibly including an Access Point (AP), within full visibility of one another, so that carrier sense is fully functional. Saturation traffic is considered, i.e. each station always has a packet to send. We characterize the service process of the network at the MAC layer, i.e. the sequence of times between two consecutive service completions. In this context, service of a MAC frame is completed when the frame is successfully delivered to its destination or when it is discarded after the maximum number of transmission attempts has been reached, as envisaged by the IEEE 802.11 DCF standard [17]. Service completion can be viewed both from an individual side (a tagged station service completion) or from a collective standpoint (service completion irrespective of the originator of the served MAC frame).

Many papers have dealt with the analysis of packet service times in IEEE 802.11 wireless networks; most of them ([4], [5], [6]) have followed a Z-transform based approach, leading to approximate expressions for the generating function of the MAC access delay. It is then possible to compute the mean, the variance and, with a numerical inversion, the distribution of the service time of MAC frames. An expression for the (global) 802.11 service time distribution has been derived in [7], by following an approach based on the system approximation technique.

We define an analytical model able to describe the service process from both points of view, i.e. the probability distribution of the service times and of the number of frames served in between two service completion epochs of a tagged station. The results of the model are shown to be quite accurate compared with simulations. Simulation results are obtained by means of an ad hoc code, implementing a full fledged version of the IEEE 802.11 DCF for a traffic saturated infra-structured IBSS with a constant number of active stations (i.e. each active station is always backlogged).

By exploiting the model we highlight that, for a given station, very large service times and bursts of interposed frames from other stations are not negligible. As a matter of example, service times larger than 1 second can be achieved with probability in the order of  $10^{-3}$ . With a similar probability, with an overall population of 15 stations *hundreds* of frames of other stations can be served in between two consecutive frames belonging to the same station. In other words there can be quite long intervals when a given backlogged station does not receive service at all: hence the burstiness.

It is known that 802.11 DCF gives a preferential treatment to stations that just transmitted successfully. In [15], the ALOHA and CSMA/CA protocols are compared from the point of view of short-term fairness. We make this notion of fairness quantitative and give analytical tools to evaluate how it affects the service offered by 802.11 DCF. We can pinpoint that a major cause of burstiness lies in the very large value of the maximum contention window as compared to the default value of the minimum one (typically, 1023 as opposed to 31). We refer to the maximum contention window as *large* because of a practical (not conceptual or theoretical) remark: a single 802.11 IBSS can hardly be conceived to offer service to more than a few tens of *simultaneous* traffic flows. Although there is no difficulty in evaluating 802.11 analytical models with up to hundreds of stations, it is very unrealistic to have so many contending, *simultaneously* active stations. Once we recognize that reasonable values of  $n$  are under a few tens, 1023 appears to be an excessive value for the maximum contention window. By exploiting the model, we evaluate the trade-off between average long term throughput and service time burstiness; we show that the latter can be significantly reduced by accepting a minor throughput degradation. It is well known that the variability of service times adversely affects queue performance of backlogged traffic inside stations (e.g., mean queue delays are proportional to the coefficient of variation of the service times). In view of supporting real time and streaming services on WLANs, an excessive service time jitter is a problem as well. Moreover burstiness in the service process can degrade TCP performance due to ACK compression.

The rest of the paper is organized as follows. In Section II

modeling assumptions are stated. The transient Markov chains of the analytical model are laid out in Section III. Section IV applies the discrete time Markov chain to the analysis of the service times and also presents numerical results. Conclusions are drawn in Section V.

## II. 802.11 DCF MARKOV MODEL

The model of 802.11 DCF is derived under the following assumptions:

- *Symmetry*: stations are statistically indistinguishable, i.e. traffic parameters (input frame rate, frame length) and multiple access parameters (e.g. maximum retry limit) are the same.
- *Proximity*: every station is within the coverage area of all others, i.e. there are no hidden nodes.
- *Saturation*: stations always have packets to send.

Along with these we introduce two simplifying hypotheses:

- *Independence*: states of different stations are realization of independent random processes.
- *Geometric Back-off*: back-off counter probability distribution is geometric ( $p$ -persistent model of the DCF, [19]).

The last two hypotheses are useful to keep the analytical model simple, hence practical. The independence hypothesis is essential to describe the system dynamics by using a low dimensionality Markov chain; its validity has been discussed from a theoretical viewpoint in [8][21] and is checked against simulations in our numerical results as well as in many other works, all showing that under traffic saturation assumption, independence based models work fine for first order metrics (e.g. throughput, mean delay). As for the geometric distribution of the back-off counter, it is only used to obtain a further simplified description of the system dynamics in terms of a Markov chain.

Both hypotheses are justified by the more than satisfactory results of the model as compared to simulations. Simulation results are obtained by means of an ad hoc simulation code that reproduces the 802.11 DCF protocol under the *Symmetry*, *Proximity* and *Saturation* hypotheses for an IBSS with a fixed number of active stations. All relevant details from standard are taken into account in the simulator. Its main purpose is to check the extent to which Independence assumption and geometric approximation provide good results when computing second order metrics (e.g. service time variance) or even probability distributions.

Thanks to the *Symmetry* assumption we focus on a tagged station and denote by  $X(t)$  its back-off stage at time  $t$ . Let  $m$  be the maximum retry number, i.e. the maximum number of transmission attempts before discarding a MAC frame. Then  $X(t) \in \{0, 1, \dots, m\}$ . Let  $\{t_k\}$  be the sequence of time instants when the back-off counter of the tagged station is decremented. Thanks to the geometric back-off distribution hypothesis,  $X_k \equiv X(t_k)$  is a Markov chain. The structure of one-step transition is as depicted in Fig. 1.

Let  $\tau_i$  denote the transmission probability in state  $i$  ( $i = 0, 1, \dots, m$ ). If  $B_i$  denotes the number of slots in a back-off time at stage  $i$ , the geometric distribution hypothesis means that  $\mathcal{P}(B_i = r) = (1 - \tau_i)^r \tau_i, r \geq 0$ ;  $\tau_i$  is found by requiring that the mean back-off at stage  $i$  be  $(W_i - 1)/2$ , i.e. the same value holding in case of a uniformly distributed back-off over

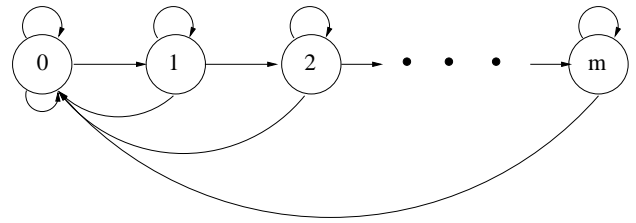


Fig. 1. 802.11 DCF Markov chain.

the interval  $[0, W_i - 1]^1$ . Then, it is  $\tau_i = 2/(W_i + 1)$ .

Non null one-step transition probabilities of the Markov chain  $X_k$  are given by:

$$\begin{aligned} \phi_{0,0} &= 1 - \tau_0 + \tau_0(1 - \tau)^{n-1} \\ \phi_{i,i} &= 1 - \tau_i \text{ for } i = 1, \dots, m \\ \phi_{i,i+1} &= \tau_i(1 - (1 - \tau)^{n-1}) \text{ for } i = 0, \dots, m-1 \\ \phi_{i,0} &= \begin{cases} \tau_i(1 - \tau)^{n-1} & \text{for } i = 1, \dots, m-1 \\ \tau_m & \text{for } i = m \end{cases} \end{aligned}$$

where  $\tau$  is the average transmission probability, i.e.  $\tau = \sum_i \tau_i \pi_i$ , where  $\pi_j$  are the steady state Markov chain probabilities. The above expressions come from the independence hypothesis. For  $i < m$  a transition from the state  $i$  to the state  $i + 1$  represents the event that the tagged station attempts to access the channel but the transmitted packet collides. If the tagged station does not attempt to access the channel, the state does not change. If the tagged station accesses the channel successfully, a transition from  $i$  to 0 occurs. If  $i = m$ , a transition from  $m$  to 0 occurs, when the tagged station attempts to access the channel (successfully or unsuccessfully).

The steady state probability  $\pi_i$  of the Markov chain are ( $i = 0, 1, \dots, m$ ):

$$\pi_i = \frac{p^i / \tau_i}{\sum_{j=0}^m p^j / \tau_j} = \frac{p^i (W_i + 1)}{\sum_{j=0}^m p^j (W_j + 1)} \quad (1)$$

where  $p = 1 - (1 - \tau)^{n-1}$  represents the tagged station collision probability conditional on transmission attempt. It is easily verified that  $\pi_i$  above coincides with the steady state probability of the tagged station staying in back off stage  $i$  as calculated from the two-dimensional Markov chain in [2]. Also the analytic expressions of the average transmission probability  $\tau$  and the average throughput are just the same for the original two-dimensional model and for our simplified one<sup>2</sup>. Further, the fixed point iteration in [20] is immediately recovered by writing  $\tau = \sum_{i=0}^m \tau_i \pi_i$ , with the  $\pi_i$ 's as given in eq. (1).

The average throughput results obtained with the above model are depicted in Figure 2, varying the number of competing stations  $n$ , for different values of  $CW_{max}$ . As far as regards other system parameters, we used standard 802.11b settings, 11 Mbps data rate, 1 Mbps basic rate and a MAC data frame payload of 1500 bytes. It is apparent that throughput degradation resulting from a smaller than standard value of  $CW_{max}$  (e.g.  $CW_{max}=255$ ) is almost negligible.

<sup>1</sup>According to the IEEE 802.11 standard,  $W_i = \min\{CW_{max}, 2^i(CW_{min} + 1) - 1\}$ , for  $i = 0, \dots, m$ , where usual values of  $CW_{min}$  and  $CW_{max}$  are 31 and 1023 respectively. The results of this paper are independent of the specific values given to the  $W_i$ 's, provided they form an increasing sequence with  $i$ .

<sup>2</sup>This is even a stronger simplification than the one in [9].

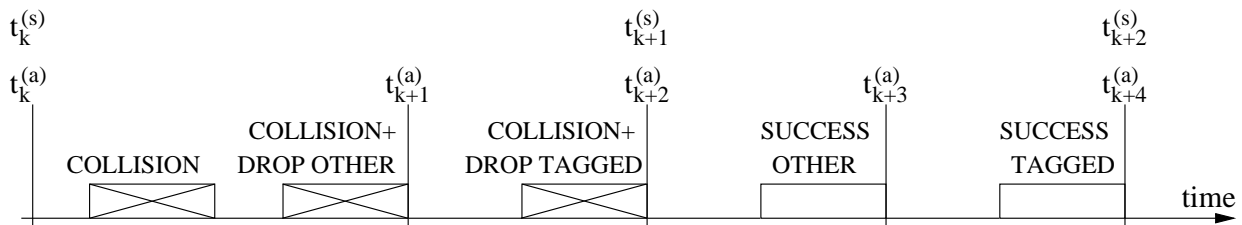
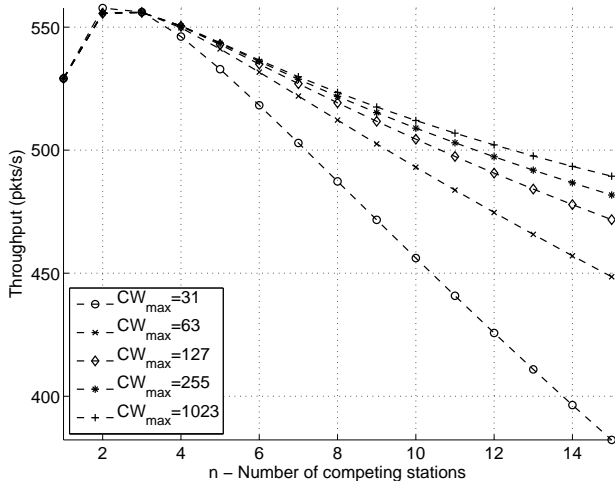


Fig. 3. 802.11 DCF medium access evolution.


 Fig. 2. Throughput varying the number of competing stations  $n$  for  $m = 7$  and different values of  $CW_{max}$ .

### III. 802.11 DCF SERVICE TRANSIENT MARKOV CHAIN

Let  $t_k$  denote the  $k$ -th back-off decrement time; it occurs either after an idle time lasting a slot time or after a transmission attempt followed by a slot time. At each transmission attempt, either a frame is successfully delivered, or a collision occurs<sup>3</sup>. In the former case, a frame has been served, i.e. we have a frame service completion epoch. In the collision case, frame delivery is attempted again after a back off time, except for those frames whose maximum number of attempts has been exhausted. For those frames service is complete as well, although ending up with a failure. Let  $t_k^{(a)}$  be the service completion epochs (either with success or failure) as seen from the overall system point of view, i.e. irrespectively of the specific station that completes its frame service; let also  $t_k^{(s)}$  be the service completion epochs (either with success or failure) as seen by a tagged single station. The sequence  $\{t_k^{(a)}\}$  is obtained by sampling the full sequence  $\{t_k\}$  and the sequence  $\{t_k^{(s)}\}$  turns out as further sampling of the sequence  $\{t_k^{(a)}\}$ . Figure 3 depicts an example of 802.11 DCF time evolution at the considered sampling points.

The  $k$ -th service completion times for the tagged station and for the collective ensemble of contending stations are denoted respectively as  $\Theta_{s,k}$  and  $\Theta_{a,k}$ , respectively. Under the traffic saturation assumption we have  $\Theta_{s,k} = t_k^{(s)} - t_{k-1}^{(s)}$  and  $\Theta_{a,k} = t_k^{(a)} - t_{k-1}^{(a)}$ ; at steady state they are distributed as a common random variable:  $\Theta_{s,k} \sim \Theta_s$  and  $\Theta_{a,k} \sim \Theta_a$ ,  $\forall k$ . In the following we develop a regenerative model of service completions allowing us to compute the statistics of  $\Theta_s$  and  $\Theta_a$ . Such models are based on a variation of the

<sup>3</sup>We assume ideal physical channel, so that no frame loss due to receiver errors takes place.

ergodic Markov chain in Section II. The basic remark is that the back-off process that rules the transmission attempts to the medium, described by the Markov chain in Figure 1, is *independent* of the time spent in a transmission attempt, under the traffic saturation hypothesis; yet *service times* do depend on the time spent in the transmission attempts, hence on MAC frame lengths and used bit rate.

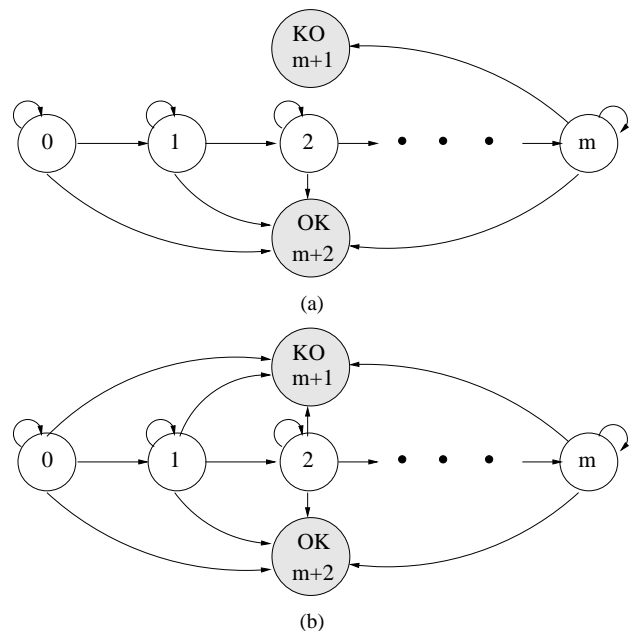


Fig. 4. Single (a) and All (b) stations transient Markov chains.

Figure 4 depicts the two transient Markov chains. The state of the Markov chain is  $Y_k \in \{0, 1, \dots, m+2\}$ , where the last two states denote failure ( $m+1 \equiv KO$ ) and frame delivery success ( $m+2 \equiv OK$ ) respectively. We are interested in a transient behavior of the chain where the initial probability vector at time 0 is  $[\alpha \ 0 \ 0]$ , where the  $(m+1)$ -dimensional row vector  $\alpha$  gives the initial probabilities of the states  $\{0, 1, \dots, m\}$  and the last two states are absorption ones<sup>4</sup>.

Let us define some notation.

- $\Phi$  = the one step Markov chain transition probability matrix.
- $\Psi$  = the  $(m+1) \times (m+1)$  substochastic submatrix of the one step Markov chain transition probability matrix relevant to the first  $m+1$  states; it has positive elements only on the diagonal and super-diagonal.
- $\varphi_{m+1}$  and  $\varphi_{m+2} = (m+1)$ -dimensional column vectors of the transition probability from each of the transient states into the absorption states  $m+1$  (i.e. failure of

<sup>4</sup>We assume initialization cannot take place directly in one of the two absorption states.

delivery, packet drop due to maximum retry limit) and  $m + 2$  (i.e. success of delivery) respectively.

- $\mathbf{D}_1 = \text{diag}[\varphi_{m+1}]$ ,  $\mathbf{D}_2 = \text{diag}[\varphi_{m+2}]$  and  $\mathbf{D} = \mathbf{D}_1 + \mathbf{D}_2$ .

We have

$$\Phi = \begin{bmatrix} \Psi & \varphi_{m+1} & \varphi_{m+2} \\ \mathbf{0} & 1 & 0 \\ \mathbf{0} & 0 & 1 \end{bmatrix} \quad (2)$$

Let  $T$  be the number of transitions to absorption, given that the initial probability distribution is  $[\alpha \ 0 \ 0]$ . Then, it can be verified that:

$$\mathcal{P}(T = t; S_T = j; A = m + k) = [\alpha \Psi^{t-1} \mathbf{D}_k]_j \quad (3)$$

$$t \geq 1, j = 0, 1, \dots, m, k = 1, 2$$

where  $S_T$  is the transient state from which absorption occurs,  $A$  is the resulting absorption state and  $[\mathbf{x}]_j$  is the  $j$ -th element of the vector  $\mathbf{x}$ . The marginal distribution of each of these variables can be obtained easily. In particular, the probability distribution of  $T$  is given by  $f_T(t) = \alpha \Psi^{t-1} \mathbf{D} \mathbf{e} = \alpha \Psi^{t-1} (\varphi_{m+1} + \varphi_{m+2})$ ,  $t \geq 1$ , where  $\mathbf{e}$  is a column vector of 1's of size  $m + 1$ .

The random variable  $T$  only counts Markov chain transitions until the service completion occurs. Service time distribution can be found by de-normalizing time, so accounting for the actual duration of the transmissions/collisions. This is done in Section IV. The rest of this Section is devoted to the complete identification of the transient Markov chains that will be exploited in Section IV. To this end there remains to identify the vector  $\alpha$  and the values of the entries of the one step transition probability matrix  $\Phi$ . Both of these quantities depend on the subset of embedded times we consider, namely  $\{t_k^{(s)}\}$  or  $\{t_k^{(a)}\}$ .

#### A. Tagged station service transient Markov chain

The Markov chain related to the time series  $\{t_k^{(s)}\}$  is depicted in Figure 4(a). The transition probability  $\varphi_{i,i}$  is the probability that the tagged station remains in state  $i$ :

$$\varphi_{i,i} = 1 - \tau_i$$

A state transition from  $i$  to  $i + 1$  and from the state  $m$  to the state  $m + 1$  (i.e. transmission failure absorption state) occurs when the tagged station attempts to access the channel, but at least one of the other stations collides with it:

$$\varphi_{i,i+1} = \tau_i (1 - (1 - \tau)^{n-1}), \quad i = 0, 1, \dots, m$$

The transition probability from state  $i$  to  $m + 2$  (i.e. transmission success absorption state) is:

$$\varphi_{i,m+2} = \tau_i (1 - \tau)^{n-1}, \quad i = 0, 1, \dots, m$$

All other entries of the matrix  $\Phi$  are null, except for  $\varphi_{m+1,m+1} = \varphi_{m+2,m+2} = 1$ . Therefore, the structure of the one step transition matrix of the transient Markov chain is as follows:

$$\Phi = \left[ \begin{array}{cccc|cc} \varphi_{0,0} & \varphi_{0,1} & \cdots & 0 & 0 & \varphi_{0,m+2} \\ 0 & \varphi_{1,1} & \cdots & 0 & 0 & \varphi_{1,m+2} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \varphi_{m,m} & \varphi_{m,m+1} & \varphi_{m,m+2} \\ \hline 0 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 1 \end{array} \right] \quad (4)$$

The initial probability vector  $\alpha$  is  $[1, 0, \dots, 0]$ , since the tagged station always restarts from the backoff stage 0, both

after a successful transmission and after a packet drop due to the maximum retry limit  $m$ .

#### B. All stations service transient Markov chain

The derivation of the Markov chain transition probabilities related to the time series  $\{t_k^{(a)}\}$  is more involved. According to the independence assumption, the states of the stations other than the tagged one are independent of one another and they are all distributed according to the ergodic probability distribution in eq. (1).

As for the expressions of the one-step transition probabilities  $\varphi_{i,j}$  of the Markov chain describing the state of the tagged station over time epochs  $\{t_k^{(a)}\}$ , we have

$$\begin{aligned} \varphi_{i,i} &= (1 - \tau_i)(1 - \tau)^{n-1} + (1 - \tau_i) \times \\ &\quad \times \sum_{k=2}^{n-1} \binom{n-1}{k} (\tau - \tau_m \pi_m)^k (1 - \tau)^{n-1-k} \\ \varphi_{i,i+1} &= \tau_i \sum_{k=1}^{n-1} \binom{n-1}{k} (\tau - \tau_m \pi_m)^k (1 - \tau)^{n-1-k} \\ &\quad i < m \\ \varphi_{i,m+1} &= \sum_{k=2}^{n-1} \binom{n-1}{k} [\tau^k - (\tau - \tau_m \pi_m)^k] \times \\ &\quad \times (1 - \tau)^{n-1-k} + \tau_i (n-1) \tau_m \pi_m (1 - \tau)^{n-2} \\ &\quad i < m \\ \varphi_{m,m+1} &= \tau_m [1 - (1 - \tau)^{n-1}] + (1 - \tau_m) \times \\ &\quad \times \sum_{k=2}^{n-1} \binom{n-1}{k} [\tau^k - (\tau - \tau_m \pi_m)^k] \times \\ &\quad \times (1 - \tau)^{n-1-k} \\ \varphi_{i,m+2} &= \tau_i (1 - \tau)^{n-1} + (1 - \tau_i)(n-1)\tau(1 - \tau)^{n-2} \end{aligned}$$

where we used the probability that  $j$  of the other stations that attempt transmission out of  $k$  do so in their last stage, namely  $\binom{k}{j} (\tau_m \pi_m)^j (\tau - \tau_m \pi_m)^{k-j}$ . All other transition probabilities not listed above are null. The overall structure of the matrix  $\Phi$  in this case is as given in eq. (4), except the first  $m$  entries of the  $(m + 1)$ -th column are all positive.

The loop transition of state  $i$  is due to no station attempting transmission or some other stations being involved in a collision (but not at their last transmission attempt) and the tagged one being idle. Transition from state  $i$  to  $i + 1$  ( $i < m$ ) is triggered by a collision involving the tagged station with no other station involved being in the last stage ( $m$ -th transmission attempt). A transition from state  $i$  to  $m + 1$  corresponds to the end of a service time with (at least one) MAC frame discard: this occurs when: i) the tagged station attempts a transmission and at least another station in the last stage transmits as well; ii) the tagged station stays idle, but a collision involving other stations occurs and at least one of them is in its last stage. A transition from state  $i$  to state  $m + 2$  corresponds to a service termination with successful MAC frame delivery. This occurs iff either the tagged station or any other station is the only one to transmit. Finally, the special case of a transition from state  $m$  to state  $m + 1$  is triggered by either the tagged station being involved in a collision or a collision involving other stations taking place, with at least one involved station in its last stage.

Also finding  $\alpha$  for the Markov chain embedded at times  $\{t_k^{(a)}\}$  is more involved, essentially because a service time

completion occurs for a MAC frame belonging either to the tagged station or to (*at least*) one of the other stations. The detailed derivation can be found in the Appendix.

#### IV. 802.11 DCF SERVICE TIME CHARACTERIZATION

We want to characterize the burstiness in the 802.11 DCF service time. Two different metrics are defined: i) the tagged station service time distribution, ii) the distribution of the number of service completions of stations other than the tagged one between two consecutive tagged station service completions. By exploiting previous models, we are able to fully characterize these issues.

##### A. Service Time Distribution

Up to now, we confined ourselves to the realm of embedded Markov epochs, to obtain the probability distribution of the absorption time  $T$  in terms of number of embedded points. If each transition takes a different time and we are interested in the overall actual time (not just number of transitions), we need to de-normalize the probability distribution of  $T$ . Let then  $f_{i,j}(s)$  be the Laplace transform of the probability density function of the time required to make a transition from state  $i$  to state  $j$  in the transient Markov chains defined in Section III and let  $\mathbf{H}(s)$  be the  $(m+3) \times (m+3)$  matrix whose entry  $g_{i,j}(s)$  is  $f_{i,j}(s)\varphi_{ij}$ ,  $i, j = 0, 1, \dots, m+2$ ; note that  $\mathbf{H}(1) = \mathbf{\Phi}$ . Let also: i)  $\mathbf{G}(s)$  be the  $(m+1) \times (m+1)$  matrix obtained from  $\mathbf{H}(s)$  by considering only the transient states; ii)  $\mathbf{D}_k(s) = \text{diag}[h_{0,m+k}(s) \ h_{1,m+k}(s) \ \dots \ h_{m,m+k}(s)]$  for  $k = 1, 2$ . Then, we can extend the result in eq. (3) to the Laplace transform of the service time probability density

$$f_{\Theta}(s; T = t, S_T = j; A = m+k) = [\alpha \mathbf{G}(s)^{t-1} \mathbf{D}_k(s)]_j \quad (5)$$

for  $t \geq 1$ ,  $j = 0, 1, \dots, m, k = 1, 2$ . The function  $f_{\Theta}(s; T = t, S_T = j; A = m+k)$  is the Laplace transform of the probability density function of the absorption time  $\Theta$  conditional on absorption in  $t$  steps, from  $j$  towards  $m+k$ , i.e. the time required to complete service of a MAC PDU in  $t$  steps of the transient Markov chain, ending up with a failure ( $k = 1$ ) or a success ( $k = 2$ ) and leaving the state of the tagged station at stage  $j$ .

The Laplace transform of the probability density function of the unconditional absorption time, i.e. the MAC frame service time  $\Theta$ , is found by summing up over  $t$ ,  $j$  and  $k$  in eq. (5). Then

$$f_{\Theta}(s) = \alpha [\mathbf{I} - \mathbf{G}(s)]^{-1} [\mathbf{D}_1(s) + \mathbf{D}_2(s)] \mathbf{e} \quad (6)$$

Let now consider the Markov chain that represents the service completion times of the tagged station, i.e. the time it takes for a tagged station frame to be successfully delivered or discarded because of exceeding the number of retransmission attempts; this the random variable denoted as  $\Theta_s$ . Then, we have  $\alpha = [1, 0, \dots, 0]$  and the entries of the matrix  $\mathbf{\Phi}$  are as in Section III-A.

The forward transitions, i.e. those from state  $k$  to state  $k+1$  ( $k = 0, 1, \dots, m-1$ ), require the time to perform a collision,  $T_c$ , which is constant if we assume a same constant data frame payload length for all stations. Therefore,  $g_{k,k+1}(s) = \varphi_{k,k+1} e^{-sT_c}$ ,  $k = 0, \dots, m-1$ .

The time of the loop transition of each transient state equals

- 1)  $\delta$ , i.e. the count-down slot time of IEEE 802.11 DCF, in case no other station attempts transmission, hence with probability  $p_e = (1 - \tau)^{n-1}$ ;

- 2)  $T_s$ , i.e. time required for a successful frame transmission and acknowledgment, if only one of the other stations transmits, hence with probability  $p_s = (n-1)\tau(1 - \tau)^{n-2}$ ;
- 3)  $T_c$ , i.e. the time it takes for a collision among other stations, in case more than one of the other stations attempt transmission, hence with probability  $p_c = 1 - p_e - p_s$ .

Therefore, the Laplace transform of the probability density of the time required for a loop transition of state  $k$  is  $\kappa(s) = [p_e e^{-s\delta} + p_s e^{-sT_s} + p_c e^{-sT_c}]$  and we have  $g_{k,k}(s) = \kappa(s)\varphi_{k,k} = \kappa(s)(1 - \tau_k)$ ,  $k = 0, \dots, m$

Finally, the time required for a transition towards the absorbing state  $m+2$  (success) is always equal to  $T_s$ ; the time of the transition to the absorbing state  $m+1$  (failure) is instead equal to  $T_c$ ; this last transition only occurs from state  $m$ .

The inverse matrix in eq. (6) can be explicitly calculated by exploiting the special structure of  $\mathbf{\Psi}$  and hence of  $\mathbf{G}(s)$ <sup>5</sup>. So we find:

$$\begin{aligned} f_{\Theta_s}(s) &= \sum_{j=0}^m \frac{e^{-sT_s} \varphi_{j,m+2} + e^{-sT_c} \varphi_{j,m+1}}{1 - \kappa(s)\varphi_{j,j}} \times \\ &\quad \times \prod_{k=0}^{j-1} \frac{e^{-sT_c} \varphi_{k,k+1}}{1 - \kappa(s)\varphi_{k,k}} \\ &= \sum_{j=0}^m e^{-s(T_s + jT_c)} (1-p) p^j \prod_{k=0}^j \frac{\tau_k}{1 - (1 - \tau_k)\kappa(s)} \\ &\quad + e^{-s(m+1)T_c} p^{m+1} \prod_{k=0}^m \frac{\tau_k}{1 - (1 - \tau_k)\kappa(s)} \quad (7) \end{aligned}$$

where  $p = 1 - (1 - \tau)^{n-1}$  is the conditional collision probability.

Moments of  $\Theta_s$  can be found by deriving  $f_{\Theta_s}(s)$ . A lengthy calculation shows that the first moment is recovered as already found in the literature, i.e.  $-f'_{\Theta_s}(0) = E[\Theta] = (1-p^{m+1})/\Lambda_1$ , where  $\Lambda_1$  is the saturation throughput of a tagged station, which is  $\Lambda_1 = \tau(1-p)/\bar{T}$ , with  $\bar{T} = \delta(1-\tau)(1-p) + T_s n \tau(1-p) + T_c [p - (n-1)\tau(1-p)]$  being the virtual slot duration [2].  $f_{\Theta_s}(s)$  can be numerically inverted by using standard methods (e.g., see [18]).

The Laplace transform of the collective service time random variable  $\Theta_a$  is very close to eq. (7), except that the expressions derived in Section III-B shall be used for  $\alpha$  and the  $\varphi_{i,j}$ 's and that the expressions of  $p_s$  and  $p_c$  appearing into  $\kappa(s)$  are different, namely  $p_s = 0$  and  $p_c = (1 - \tau_m \pi_m)^{n-1} - (1 - \tau)^{n-1} - (n-1)(\tau - \tau_m \pi_m)(1 - \tau)^{n-2}$ . The major difference is that a double summation appears, since the entries of  $\alpha$  are

<sup>5</sup> Let  $\mathbf{C}$  be a matrix whose non-null entries are only the diagonal elements  $c_{k,k} = a_k$ ,  $k = 0, 1, \dots, m$  and super-diagonal ones  $c_{k,k+1} = b_k$ ,  $k = 0, 1, \dots, m-1$ . Let also  $\mathbf{D} = \mathbf{C}^{-1}$ ;  $\mathbf{D}$  is an upper triangular matrix, whose diagonal elements are the reciprocals of the elements on the diagonal of  $\mathbf{C}$ . It can be verified that  $d_{k,j} = -b_k d_{k+1,j}/a_k$ ,  $k = 0, \dots, j-1$ ,  $j = 1, \dots, m$  and  $d_{j,j} = 1/a_j$ ,  $j = 0, 1, \dots, m$ . This yields an explicit expression for non-null entries of  $\mathbf{D}$ :

$$d_{k,j} = (-1)^{j-k} \frac{1}{a_j} \prod_{r=k}^{j-1} \frac{b_r}{a_r} \quad k = 0, 1, \dots, j; j = 0, 1, \dots, m$$

where the product reduces to 1 in case the lower range index is less than the upper one ( $\prod_{r=j}^j \equiv 1$ ).

in general positive:

$$f_{\Theta_a}(s) = \sum_{j=0}^m \frac{e^{-sT_s} \varphi_{j,m+2} + e^{-sT_c} \varphi_{j,m+1}}{1 - \kappa(s) \varphi_{j,j}} \times \sum_{i=0}^m \alpha_i \prod_{k=i}^{j-1} \frac{e^{-sT_c} \varphi_{k,k+1}}{1 - \kappa(s) \varphi_{k,k}}$$

where the  $\varphi_{i,j}$ 's are as given in Sec. III-B.

### B. Model validation

We compare service time distribution and variance obtained from the model to simulation results.

In the following numerical example we assume a data MAC frame payload of 1500 bytes, data rate = 11 Mbps, basic rate (preamble and PLCP header) equal to 1 Mbps,  $\delta=20 \mu s$ ; by using the IEEE 802.11b DCF standard values, it turns out that  $T_c=T_s=1.589 ms$  for 1500 bytes data frame payload; the contention windows are set according to the standard with  $CW_{max} = 1023$  and  $CW_{min} = 31$ .

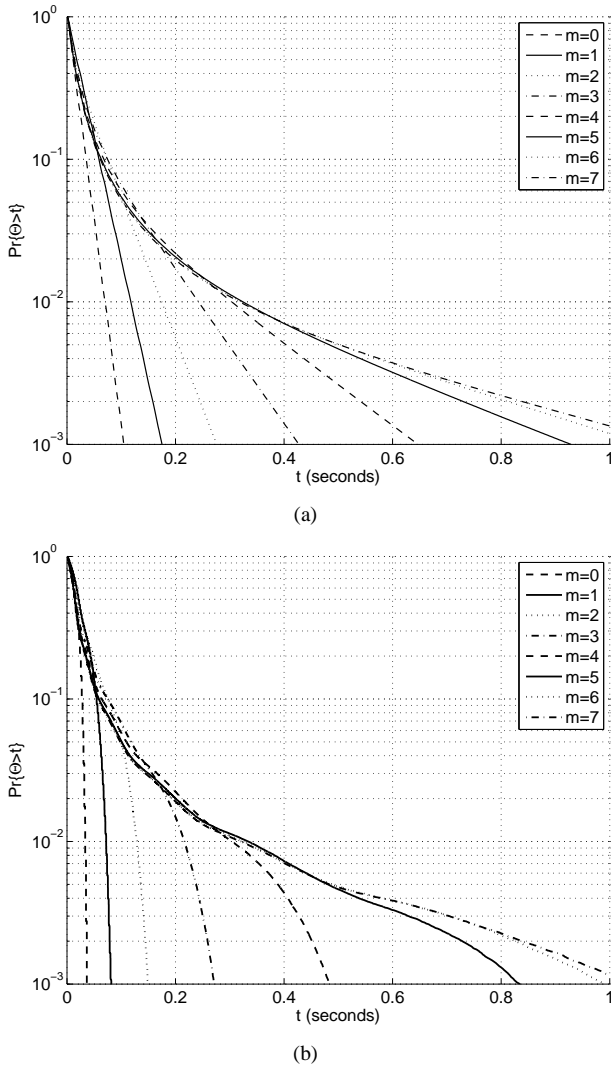


Fig. 5. Analytical (a) and Simulative (b) CCDF of the service delay for  $n=15$  and  $CW_{max} = 1023$  varying the maximum retry limit  $m$ .

Figure 5 depicts the complementary cumulative distribution function (CCDF) of the service delay obtained by inverting eq. (7) (Figure 5(a)) and the empirical CCDF obtained by means of simulations (Figure 5(b)) as a function of  $m$ , when the number of competing stations  $n$  is 15. When the max retry

limit is low, the model is not able to reproduce successfully the service delay statistics for probability values below about 0.1, since the impact of the geometric backoff assumption dominates the delay statistics. When  $m$  increases, the model reproduces successfully the CCDF. From the analysis of the CCDF, we note that the variability of the service time is quite high. E.g, when  $m$  is 7 (standard 802.11 retry limit), 1 packet over 1000 experiences a delay higher than 1 second<sup>6</sup> indicating a high level of dispersion of the service delay.

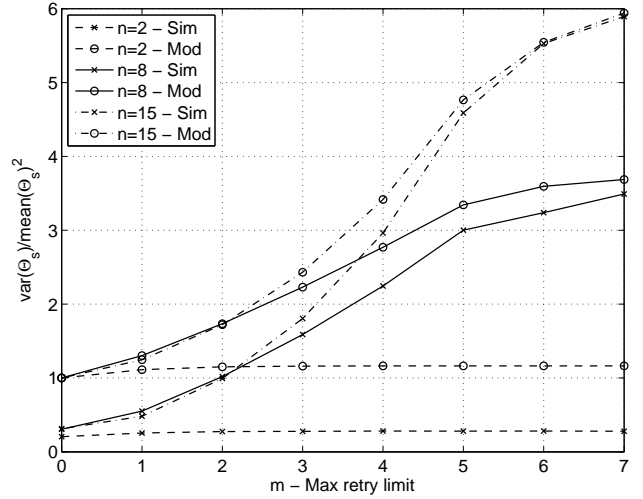


Fig. 6. Coefficient of variation of the service time,  $\mu$ : validation against simulative results.

Figure 6 depicts the ratio  $\mu$  between the variance of  $\Theta_s$  and the squared mean of  $\Theta_s$  obtained by means of eq. (7) and as derived from simulation results;  $\mu$  is a good indication of the dispersion degree of the service delay with respect to the mean service delay. Even in this case, discrepancies between model and simulation results are due to the geometrical distribution assumption that leads to an overestimation of the variance of the service time. The error vanishes as the maximum retry limit gets closer to realistic values (standard default is  $m=7$ ), except in the extreme case  $n = 2$ , where the independence assumption introduces a bias in the model results.

The analytical model is valuable thanks to its very fast computation times (orders of magnitude less than simulation times) and since it is very accurate just in those cases where it is practically useful to carefully engineer the wireless access, i.e. for a non negligible number of stations (larger than a few units) and for retry limit close to the standard value.

### C. Service times burstiness

As a further step, we characterize the burstiness of the service process. We exploit the Markov chain related to the time series  $\{t_k^{(a)}\}$  to evaluate the distribution of the number of service completions of stations other than the tagged one between two successive time epochs of the sequence  $\{t_k^{(s)}\}$ , i.e. two successive tagged station service completions. To this end, we use the transient Markov chain that describes all service completions (see Figure 4(b)). hence the entries of matrix  $\Phi$  given in Section III-B.

Let  $\mathbf{D}_o = \text{diag}[\varphi_1^{(o)} + \varphi_2^{(o)}]$  and  $\mathbf{D}_s = \text{diag}[\varphi_1^{(s)} + \varphi_2^{(s)}]$  two diagonal matrices, with vectors  $\varphi_k^{(x)}$  as defined in the

<sup>6</sup>To be compared with the average service time of a station alone, equal to 1.9 ms with the assumed parameter values.

Appendix, with  $k = 1, 2$  and  $x = o, s$ . Note that  $\mathbf{D}_o \mathbf{e} + \mathbf{D}_s \mathbf{e} = \mathbf{e} - \Psi \mathbf{e}$ . The  $(i, j)$ -th entry of  $(\mathbf{I} - \Psi)^{-1} \mathbf{D}_o$  is the probability that any station other than the tagged one terminates its service leaving the tagged station in state  $j$ , conditional on the tagged station starting out in state  $i$ ; this is just the one-step probability transition matrix of the tagged station phase (state) on a service completion by another station.

The probability that  $k$  services of other stations occur before the tagged station is served, i.e. between two services of the tagged station, is given by:

$$q_k = \alpha [(\mathbf{I} - \Psi)^{-1} \mathbf{D}_o]^k (\mathbf{I} - \Psi)^{-1} \mathbf{D}_s \mathbf{e}, \quad k \geq 0 \quad (8)$$

with  $\alpha = [1 \ 0 \ \dots \ 0]$ .

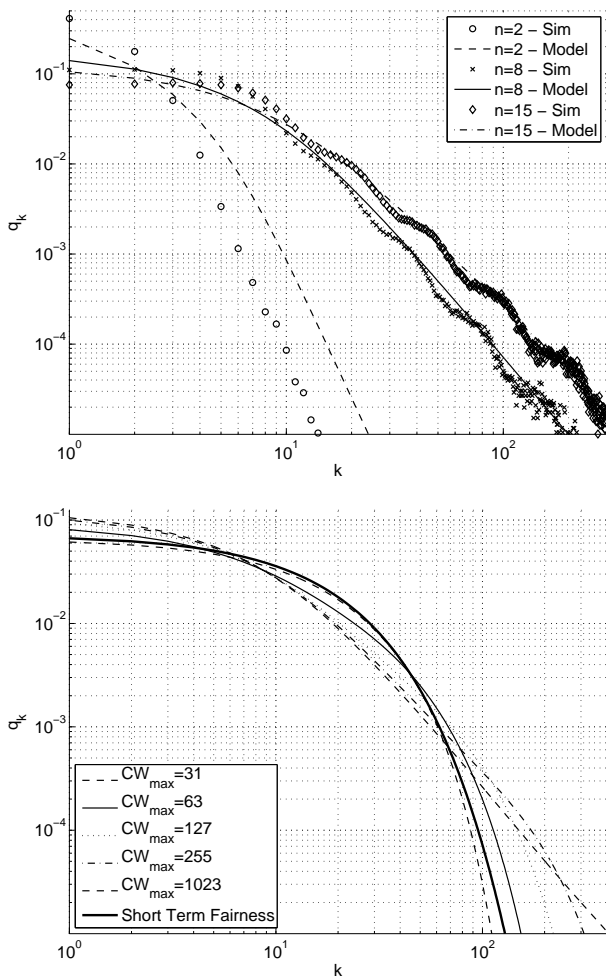


Fig. 7. Probability  $q_k$  that  $k$  services of other stations occur between two consecutive services of the tagged station: (a) comparison between model and simulations; (b) model results for various values of  $CW_{max}$ .

Figure 7(a) depicts  $q_k$  derived from the analytical model against simulation results, for different values of  $n$  and  $m = 7$ . When the number of competing stations  $n$  is large enough, the model is able to reproduce the burstiness level of the service process. When  $n$  is 2, the model does not correctly reproduce  $q_k$ ; in this scenario, the independence hypothesis does not hold, since the two competing stations' evolutions are correlated. While this does not significantly affect the estimate of the throughput, it turns out to be more critical in the case of second order or distribution tail evaluation. Accuracy is recovered for larger values of  $n$  (e.g. in the order of 10).

Figure 7(b) depicts  $q_k$  obtained by means of the analytical model, for different values of  $CW_{max}$ , for  $m = 7$  and

$n = 15$ . Moreover the solid bold line depicts  $q_k$  when using a short term fair random scheduler, i.e. a random scheduler that chooses the next served station independently of previous served stations with the same probability; in this case  $q_k = (1 - 1/n)^k 1/n$ . The heavier right tail of the 802.11 scheduling distribution with large values of  $CW_{max}$  highlights the burstiness of the 802.11 service process. Such burstiness can be reduced by choosing a smaller value of  $CW_{max}$ ; in this case the figure highlights that the 802.11 DCF behaves as a short term fair scheduler.

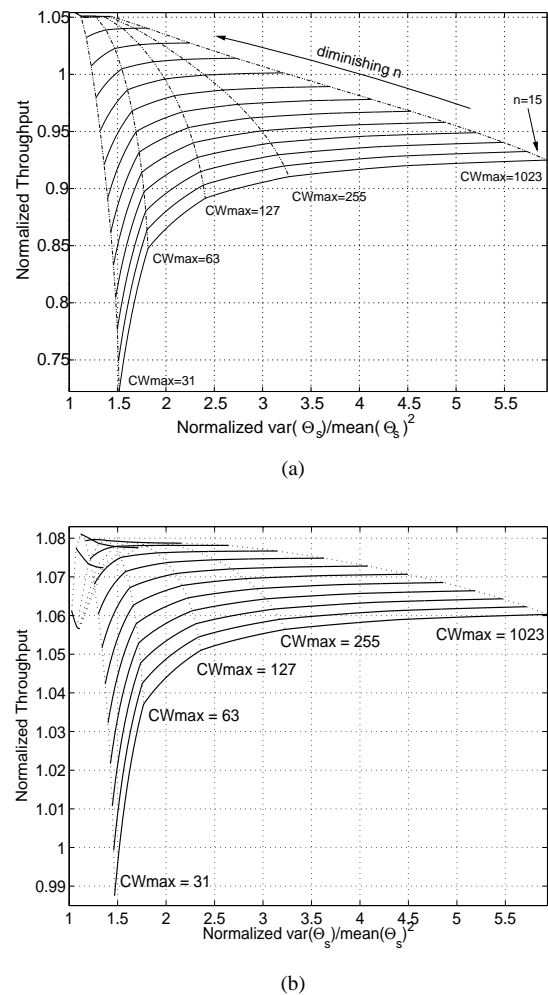


Fig. 8. Squared coefficient of variation of the service time,  $\mu$ : tradeoff against normalized throughput - Basic access (a) and RTS/CTS (b).

The impact of a reduction of  $CW_{max}$  on the system throughput is investigated in Figure 8, where we plot the trade-off between performance penalty and service time jitter, for both the Basic Access (Figure 8(a)) and RTS/CTS (Figure 8(b)) access methods. Performance is measured as long term average throughput normalized with respect to throughput value with  $n = 1$  and  $CW_{max} = 1023$ . Jitter is measured by the squared coefficient of variation of the service time  $\mu$  normalized with respect to the value of  $\mu$  in case  $n = 1$  and  $CW_{max} = 1023$ . The maximum contention window  $CW_{max}$  varies from 31 to 1023, and the number of active terminals,  $n$ , varies from 2 to 15. Dashed lines through the graphs join together points where  $CW_{max}$  has the same value, from 31 to 1023. The key result is that most of the right portion of the curves is almost flat, pointing out that a substantial reduction of the service time jitter can be achieved in spite of

a minor throughput degradation. By decreasing  $CW_{max}$ , it is possible to limit the dispersion of the service delay around the mean service delay, without significantly reducing the mean throughput. As a matter of example, using  $CW_{max}=127$  instead of 1023 when  $n$  is 15, the delay dispersion is more than halved against a throughput reduction of about 4% for the basic access method, and of less than 1% for the RTS/CTS access method.

Until now we have only shown results obtained for a data rate of 11 Mbps and a MAC frame payload of 1500 bytes. We also evaluated the performance of the system and the service time jitter for different payload sizes, and for some of the data rates specified by the 802.11b and 802.11g standards<sup>7</sup>. Table I shows the values of the long term average throughput, normalized to the data rate, for both the Basic Access (Table I(a)) and the RTS/CTS (Table I(b)) access methods. We set  $n = 15$  and  $m = 7$ .

Results show that RTS/CTS performs better than the basic access only for very large payloads and low data rates. The coefficient of variation of the (individual) service times is tabulated in Table II; the access delay jitter appears to be almost independent of the data rate and packet size.

TABLE I  
NORMALIZED THROUGHPUT ( $n = 15; m = 7$ )

(a) Basic Access			
Frame payload (bytes)	Data rate (Mbps)		
	2	11	54
100	0.292	0.096	0.059
500	0.591	0.323	0.228
1500	0.712	0.534	0.434
4000	0.761	0.671	0.606

(b) RTS/CTS			
Frame payload (bytes)	Data rate (Mbps)		
	2	11	54
100	0.223	0.062	0.044
500	0.590	0.249	0.188
1500	0.812	0.499	0.410
4000	0.920	0.726	0.649

## V. FINAL REMARKS

A new approach for the derivation of the service time distribution is defined in this work. It allows a full characterization of service times of a tagged station and of the system as a whole, i.e. inter-departure times between packets belonging to a same station or to any station. The model is exploited to highlight burstiness of service times and the key system parameters it depends upon, in particular the maximum value of the contention window of IEEE 802.11 DCF,  $CW_{max}$ . A trade-off between throughput efficiency and service time variance is quantified and discussed, showing that a minor loss on throughput can bring about a major benefit on service time smoothness, at least for practical values of the number of simultaneously competing station (e.g. less than 15).

A number of extension of this work can be envisaged. The new analytical approach lends itself to dealing also with

<sup>7</sup>For the data rates of 2 Mbps and 11 Mbps the basic bit rate (i.e., the bit rate for the preamble and PLCP header) was set to 1 Mbps, while for the data rate of 54 Mbps the basic rate was set to 6 Mbps.

TABLE II  
COEFFICIENT OF VARIATION OF THE SERVICE TIMES ( $n = 15; m = 7$ )

(a) Basic Access			
Frame payload (bytes)	Data rate (Mbps)		
	2	11	54
100	2.439	2.442	2.458
500	2.436	2.440	2.452
1500	2.435	2.437	2.446
4000	2.435	2.436	2.440

(b) RTS/CTS			
Frame payload (bytes)	Data rate (Mbps)		
	2	11	54
100	2.437	2.439	2.452
500	2.435	2.438	2.449
1500	2.434	2.436	2.444
4000	2.433	2.434	2.440

variable data frame sizes and transmission rates, i.e.  $T_s$  and  $T_c$  can be modeled as random variables. Asymptotic tail analysis for the service time survivor function can be done based on the analytical expression of the Laplace-Stieltjes transform, so as to derive a simple exponential-like approximation of the service times; also an estimation of the error introduced by the geometric back-off distribution assumption we made as opposed to uniform distribution would be useful (work is in progress on these last two topics).

More elaborate extensions should aim to relax saturated traffic assumption or equivalently to model a changing number  $n$  of backlogged stations over time; we also aim to develop a full-fledged model of the 802.11 DCF as a packet scheduler, to be used for studying performance of upper layer protocols (e.g. TCP).

A different line of application of this result (and analogous ones in the literature) is in fine tuning of contention window values, so as to control not only first order metrics (e.g. throughput) but also second order ones (e.g. variance of the service times) or even quantiles or tails of relevant performance metrics.

## APPENDIX

In the following we show how to derive  $\alpha$  for the Markov chain embedded at times  $\{t_k^{(a)}\}$ . The vector  $\varphi_{m+k}$  can be split into two arrays, each containing the transition probability towards the absorbing state  $m+k$  corresponding to service completions of the tagged station or of any other station ( $k = 1, 2$ ). Let the two components be  $\varphi_k^{(s)}$  and  $\varphi_k^{(o)}$  for the transition probabilities corresponding to service completions of the tagged station and of any other station, respectively. So,  $\varphi_{i,m+k} = \varphi_{i,k}^{(s)} + \varphi_{i,k}^{(o)}$ ,  $i = 0, 1, \dots, m; k = 1, 2$ . It can be found by starting from the expression for the  $\varphi_{i,m+k}$ 's that

$$\varphi_{i,1}^{(s)} = 0 \quad i = 0, 1, \dots, m-1$$

$$\begin{aligned} \varphi_{m,1}^{(s)} &= \tau_m \sum_{k=1}^{n-1} \binom{n-1}{k} (1-\tau)^{n-1-k} \times \\ &\times \sum_{j=0}^k \binom{k}{j} \frac{1}{1+j} (\tau_m \pi_m)^j (\tau - \tau_m \pi_m)^{k-j} \end{aligned}$$

$$\begin{aligned}\varphi_{i,1}^{(o)} &= (1 - \tau_i)[1 - (1 - \tau_m \pi_m)^{n-1} \\ &\quad - (n-1)\tau_m \pi_m (1 - \tau)^{n-2}] \\ i &= 0, 1, \dots, m-1\end{aligned}$$

$$\begin{aligned}\varphi_{m,1}^{(o)} &= (1 - \tau_m)[1 - (1 - \tau_m \pi_m)^{n-1} \\ &\quad - (n-1)\tau_m \pi_m (1 - \tau)^{n-2}] + \\ &\quad + \tau_m \sum_{k=1}^{n-1} \binom{n-1}{k} (1 - \tau)^{n-1-k} \times \\ &\quad \times \sum_{j=0}^k \binom{k}{j} \frac{j}{1+j} (\tau_m \pi_m)^j (\tau - \tau_m \pi_m)^{k-j}\end{aligned}$$

$$\varphi_{i,2}^{(s)} = \tau_i (1 - \tau)^{n-1} \quad i = 0, 1, \dots, m$$

$$\varphi_{i,2}^{(o)} = (1 - \tau_i)(n-1)\tau(1 - \tau)^{n-2} \quad i = 0, 1, \dots, m$$

The case of joint service completion (with failure) has been arbitrarily split among the tagged station and other stations ending their packet service time, by assigning the service completion event to any of the  $j+1$  stations completing service simultaneously with uniform probability.

We define  $\mathbf{D}^{(o)} = \text{diag}[\varphi_1^{(o)} + \varphi_2^{(o)}]$  and  $\mathbf{D}^{(s)} = \text{diag}[\varphi_1^{(s)} + \varphi_2^{(s)}]$ . By definition we have  $\mathbf{D}^{(o)}\mathbf{e} + \mathbf{D}^{(s)}\mathbf{e} + \Psi\mathbf{e} = \mathbf{e}$ .

Let  $\beta^{(o)} = \alpha(\mathbf{I} - \Psi)^{-1}\mathbf{D}^{(o)}\mathbf{e}$  and  $\beta^{(s)} = 1 - \beta^{(o)} = \alpha(\mathbf{I} - \Psi)^{-1}\mathbf{D}^{(s)}\mathbf{e}$  be the probabilities of a service completion of other stations and of the tagged station, respectively,  $\mathbf{I}$  being the identity matrix of size  $(m+1) \times (m+1)$ .

The probability distribution of the exit state into absorption conditional on other than tagged station performing a service completion is  $\mathbf{q}^{(o)} = \alpha(\mathbf{I} - \Psi)^{-1}\mathbf{D}^{(o)}/\beta^{(o)}$ .

To define a regenerative process, we require that

$$\begin{aligned}\alpha &= \beta^{(o)}\mathbf{q}^{(o)} + \beta^{(s)}\mathbf{e}_1 \\ &= \alpha(\mathbf{I} - \Psi)^{-1}\mathbf{D}^{(o)} + (1 - \alpha(\mathbf{I} - \Psi)^{-1}\mathbf{D}^{(o)}\mathbf{e})\mathbf{e}_1\end{aligned}\quad (9)$$

where  $\mathbf{e}_1 = [1, 0, \dots, 0]$ . By the change of variable  $\mathbf{u} = \alpha(\mathbf{I} - \Psi)^{-1}$ , we obtain  $\mathbf{u}(\mathbf{I} - \Psi) = \mathbf{u}\mathbf{D}^{(o)} + (1 - \mathbf{u}\mathbf{D}^{(o)}\mathbf{e})\mathbf{e}_1$ . From this it can be found that  $\mathbf{u} = (1 - \mathbf{u}\mathbf{D}^{(o)}\mathbf{e})\mathbf{e}_1(\mathbf{I} - \Psi - \mathbf{D}^{(o)})^{-1}$  and hence

$$\mathbf{u}\mathbf{D}^{(o)}\mathbf{e} = \frac{\mathbf{e}_1(\mathbf{I} - \Psi - \mathbf{D}^{(o)})^{-1}\mathbf{D}^{(o)}\mathbf{e}}{1 + \mathbf{e}_1(\mathbf{I} - \Psi - \mathbf{D}^{(o)})^{-1}\mathbf{D}^{(o)}\mathbf{e}}$$

hence

$$\alpha = \frac{\mathbf{e}_1(\mathbf{I} - \Psi - \mathbf{D}^{(o)})^{-1}(\mathbf{I} - \Psi)}{1 + \mathbf{e}_1(\mathbf{I} - \Psi - \mathbf{D}^{(o)})^{-1}\mathbf{D}^{(o)}\mathbf{e}}\quad (10)$$

Equation (10)<sup>8</sup> gives the expression of the initial state distribution to be used with the Markov chain of service completion of any station (either the tagged or not).

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<sup>8</sup>The numerator of eq. (10) is non negative, since the matrix multiplying  $\mathbf{e}_1$  is equal to  $\mathbf{X} = [\mathbf{I} - (\mathbf{I} - \Psi)^{-1}\mathbf{D}^{(o)}]^{-1}$ ; from the identity  $\mathbf{D}^{(o)}\mathbf{e} + \mathbf{D}^{(s)}\mathbf{e} = \mathbf{e} - \Psi\mathbf{e}$  and the positivity of the diagonal elements of  $\mathbf{D}^{(s)}$ , we get  $\mathbf{D}^{(o)}\mathbf{e} < (\mathbf{I} - \Psi)\mathbf{e}$ , i.e. the matrix  $(\mathbf{I} - \Psi)^{-1}\mathbf{D}^{(o)}$  is sub-stochastic, hence  $\mathbf{X}$  is non-negative.



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